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Peristaltic Transport of a Conducting Jeffrey Fluid in a Vertical Annulus

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Abstract

This paper is aimed to study the effects of heat transfer and magnetic field on the peristaltic flow of a Jeffrey fluid in a vertical annulus with zero Reynolds number and long wavelength approximations. The inner tube is considered to be uniform and rigid where as the outer tube is considered to have a sinusoidal wave travelling along its wall. A uniform transverse magnetic field of strength B_0 is applied perpendicular to the flow direction. Wave frame of reference moving with the velocity of the wave is used to investigate the flow. Expressions are obtained for temperature, velocity, stream function, pressure rise and frictional forces. The features of the flow characteristics are analyzed by plotting graphs and the results are discussed in detail. It is observed that the pressure rise increases with increase in Jeffrey parameter λ_1 .

Keywords :Peristaltic flow, Heat transfer, Magnetic field, Jeffrey parameter λ_1 .

Introduction

In living bodies, heat transfer plays an important role in many physiological processes as it affects the temperature and its spatial distribution in tissues. Advances in the research of bio-heat transfer paved the way to the developments in diverse areas such as hyperthermia cancer therapy, thermal diagnosis, cryogenic surgery etc. The prerequisite for further quantitative and accurate analysis of bio-heat transfer is to effectively understand and model the transfer mechanism of mass and energy in the biological system.

Thermal energy transfer in living tissues is a complex process which includes conduction, convection, radiation, interior metabolism, evaporation, phase change, and inherent temperature regulation. Furthermore, heterogeneity of living tissues and discrepancies in biological materials bio-thermal behavior have a remarkable effect of blood perfusion on the temperature field in body, which varies among different tissues and organs. Therefore, it is very difficult to build generally applicable models to precisely describe the heat transfer process, and most of the proposed bio-heat equations are very complicated. Generally, the complexity of the bio-heat transfer equations makes it difficult to obtain their analytic solutions. However, analytical solutions of these equations, if attainable, are of important significance in the study of bio-heat transfer because they can not only accurately reflect the actual physical feature of equations but also be used as

standards to verify the corresponding results of numerical calculation.

Heat transfer analysis can be used to obtain the properties of tissues such as the flow rate of blood. One such technique is thermal dilution technique. In this technique, heat is either injected or generated locally and the thermal clearance is monitored. With the knowledge of initial thermal condition and the thermal clearance rate, it is possible to estimate the blood flow rates [1]. The research on the bio heat transfer has led to many developments in medicine such as hyperthermia in which heat and cryosurgery in which coldness are used to destroy undesirable tissues such as cancer, radiation treatment in which lasers are used for surgery. In case hyperthermia it is observed that tissue can be destroyed when heated to 42-45° C [2]. Radiofrequency therapy is useful to treat more diseases such as the tissue coagulation, the primary liver cancer, the lung cancer and the reflux of stomach acid. All this has given an additional thrust for research in thermal modeling in human tissues.

Peristaltic pumping is a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of the tube in which fluid is flowing. It is a natural property observed in many biological systems which transports bio fluids. Some of the examples are transport of urine from kidney to bladder, flow of blood in smaller blood vessels, intra-uterine fluid motion etc. The transport of water and

minerals from ground to all parts of a plant is believed to be explained by peristalsis and free convection. The diameter of plants trunks are found to be varying minutely with time which justifies the application of peristalsis to this transport. The transport of water involves its movement through a porous matrix of the tree. Aikman and Anderson [3] and Canny and Phillips [4] have studied the transport of water in plants. Radhakrishnamurthy et al. [5] and Vajravelu and Radhakrishnamacharya, [6] have investigated flow through vertical porous tube with peristalsis and heat transfer. Recently Mekheimer and Abd Elmaboud [7] have studied the influence of heat transfer and magnetic field on peristaltic transport of a Newtonian fluid in a vertical annulus.

The mechanism of peristaltic transport is widely used in industrial applications like sanitary fluid transport, transport of corrosive fluid where the contact of the fluid with the machinery parts is prohibited.

A lot of literature is available both analytical and experimental on peristalsis involving viscous fluid under one or more assumptions such as long wavelength, small Reynolds number, small amplitude ratio, small wave number [8-20]. The magnetohydrodynamic peristaltic flow is of importance as it applies to many real time applications such as blood and blood pump machines. The effect of magnetic field on blood flow has been discussed by Stud et al [21], Agrawal and Anwaruddin [22], Hyat et al [23] and Mekheimer [24]. Similarly a lot of literature is available on peristaltic transport of non-Newtonian fluid [25-33] which have promising applications in physiology as many bio fluids can be modeled as non-Newtonian fluids such as Jeffrey fluids, power law fluids, Carreau fluids etc.

With all the above discussions in focus, in this work, it is proposed to study the effects of heat transfer and magnetic field on peristaltic transport of a Jeffrey fluid in a vertical porous annulus. The inner tube is rigid and the outer tube has a wave train moving independently. The flow analysis is developed in wave frame of reference moving with the velocity of the wave. The problem is solved for velocity, stream function, axial pressure gradient and frictional forces

Formulation of Problem

Consider the magnetohydrodynamic flow of a viscous, incompressible and electrically conducting fluid through the gap between two co-axial vertical tubes. The inner tube is rigidly maintained at a temperature T_1 and the outer as a sinusoidal wave travelling down its wall and it is exposed to the temperature T_0 . The geometry of the wall surface is described as (see Fig No. 1).

$$R_1' = a_1 \tag{1}$$

$$R_2' = a_2 + b \cos \frac{2\pi}{\lambda} (Z' - ct') \tag{2}$$

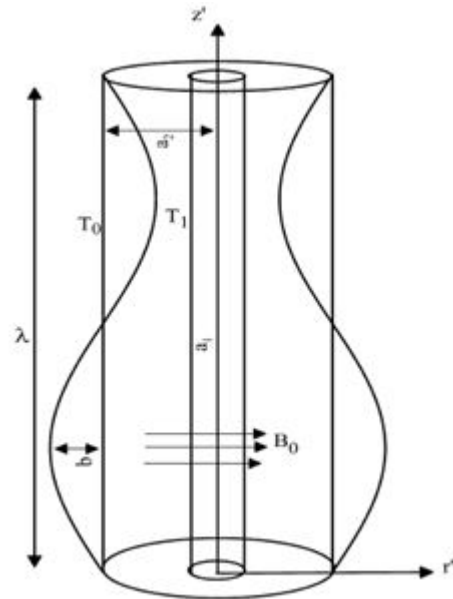


Fig.1. Geometry of problem

where a_1 , a_2 are the radii of the inner and the outer tubes, b is the amplitude of the wave, λ is the wave length. c is the wave speed and t' is the time.

A uniform magnetic field B_0 is applied transversely to the flow. The magnetic Reynolds number is small and so the induced magnetic field is negligible.

Introducing the wave frame (r', z') moving with the velocity C away from the fixed frame (R', Z') by transformation.

$$z' = Z' - ct', \quad r' = R', \quad w' = W' - c, \quad u' = U' \tag{3}$$

where (u', w') and (U', W') are the velocity components in the wave and fixed frame respectively. After using these transformations the equation of motion are

$$\frac{\partial u'}{\partial r'} + \frac{\partial w'}{\partial z'} + \frac{u'}{r'} = 0 \tag{4}$$

$$\rho \left[u' \frac{\partial u'}{\partial r'} + w' \frac{\partial u'}{\partial z'} \right] = - \frac{\partial p'}{\partial r'} + \frac{\mu}{1 + \lambda_1} \left[\frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'} \frac{\partial u'}{\partial r'} + \frac{\partial^2 u'}{\partial z'^2} - \frac{u'}{r'^2} \right] \tag{5}$$

$$\rho \left[u \frac{\partial w'}{\partial r'} + w' \frac{\partial w'}{\partial z'} \right] = -\frac{\partial p'}{\partial z'} + \frac{\mu}{1+\lambda_1} \left[\frac{\partial^2 w'}{\partial r'^2} + \frac{1}{r'} \frac{\partial w'}{\partial r'} + \frac{\partial^2 w'}{\partial z'^2} \right] - \sigma B_0^2 (w' + c) + \rho g \alpha (T' - T_0) \quad (6)$$

The energy equation is

$$\rho c_p \left[u \frac{\partial T'}{\partial r'} + w' \frac{\partial T'}{\partial z'} \right] = k \left[\frac{\partial^2 T'}{\partial r'^2} + \frac{1}{r'} \frac{\partial T'}{\partial r'} + \frac{\partial^2 T'}{\partial z'^2} \right] + Q_0 \quad (7)$$

where u' and w' are the velocity components in the r' and z' directions, respectively, ρ is the density, p' is the pressure, μ is the viscosity, T' is the temperature, α is the coefficient of linear thermal expansion of the fluid, k denotes thermal conductivity, C_p denotes specific heat at constant pressure and Q_0 is the constant heat addition /absorption. We introduce the following non-dimensional variables.

$$r = \frac{r'}{a_2}, z = \frac{z'}{\lambda}, w = \frac{w'}{c}, u = \frac{\lambda u'}{a_2 c},$$

$$p = \frac{a_2^2}{\lambda \mu c} p', r_1 = \frac{r_1'}{a_2} = \varepsilon, \delta = \frac{a_2}{\lambda},$$

$$r_2 = \frac{r_2'}{a_2} = 1 + \phi \cos(2\pi z), Re = \frac{\rho c a_2}{\mu},$$

$$\varepsilon = \frac{a_1}{a_2}, M = \sqrt{\frac{\sigma}{\mu}} a_2 B_0,$$

$$Gr = \frac{g \alpha a_2^3 (T_1 - T_0)}{v^2}, \beta = \frac{a_2^2 Q_0}{k(T_1 - T_0)},$$

$$Pr = \frac{\mu c_p}{k}, \theta = \frac{T' - T_0}{T_1 - T_0}, \quad (8)$$

where ϕ is the amplitude ratio, Re is the Reynolds number, δ is the dimensionless wave number, M is the magnetic parameter, Gr is the Grashof number, Pr is the Prandtl number and β is the non-dimensional heat source/sink parameter and dashes (') are dropped.

To proceed, we non-dimensionalize the equations (4)-(7). This yields

$$\frac{1}{r} \frac{\partial (ru)}{\partial r} + \frac{\partial w}{\partial z} = 0 \quad (9)$$

$$Re \delta^3 \left[u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right] = -\frac{\partial p}{\partial r} + \delta^2 \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (ru)}{\partial r} \right) + \delta^4 \frac{\partial^2 u}{\partial z^2} \quad (10)$$

$$Re \delta \left[u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right] = -\frac{\partial p}{\partial z} + \frac{1}{(1+\lambda_1)r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) + \delta^2 \frac{\partial^2 w}{\partial z^2} - M^2 (w+1) + Gr \theta \quad (11)$$

$$Re Pr \delta \left[u \frac{\partial \theta}{\partial r} + w \frac{\partial \theta}{\partial z} \right] = \frac{\partial^2 \theta}{\partial z^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \delta^2 \frac{\partial^2 \theta}{\partial z^2} + \beta \quad (12)$$

Using the Long wave approximation and dropping terms of order δ and higher, it follows from equations (9)-(12) that the appropriate equations describing the flow in the wave frame are

$$\frac{1}{r} \frac{\partial (ru)}{\partial r} + \frac{\partial w}{\partial z} = 0, \quad (13)$$

$$\frac{\partial p}{\partial r} = 0, \quad (14)$$

$$\frac{\partial p}{\partial z} = \left(\frac{1}{1+\lambda_1} \right) \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) - M^2 (w+1) + Gr \theta, \quad (15)$$

$$0 = \frac{\partial^2 \theta}{\partial z^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \beta, \quad (16)$$

Equation (14) shows that p is not a function of r . The corresponding dimensionless boundary conditions are

$$w = -1, \theta = 1, \text{ at } r = r_1 = \varepsilon,$$

$$w = -1, \theta = 0, \text{ at } r = r_1 \quad (17)$$

The temperature equation (16) with the boundary conditions yields

$$\theta = \frac{1}{4 \log \frac{r_1}{r_2}} \left[\left(4 + \beta (r_1^2 - r_2^2) \right) \log r - \beta (r^2 - r_2^2) \log r_1 + \left(\beta (r^2 - r_2^2) - 4 \right) \log r_2 \right] \quad (18)$$

The expression for the velocity of the fluid, obtained as a solution of equation (15) after substituting equation (18) subject to the boundary conditions (17) is given by

$$w(r, z) = \frac{1}{N^2} \left\{ \frac{d p_1}{d x} \left(\frac{C_{14} I_0(Nr) + C_{15} K_0(Nr)}{C_{11}} - 1 \right) + \frac{G (I_0(Nr) K_0(Nr_2) - K_0(Nr) I_0(Nr_2))}{C_{11}} - N^2 \right\}$$

$$+ \frac{G}{4C_{13}} \left[\left(4 + \beta (r_1^2 - r_2^2) \right) \log r - \beta (r^2 - r_2^2) \log r_1 + \left(\beta (r^2 - r_2^2) - 4 \right) \log r_2 \right] \quad (19)$$

where I_0, K_0 are the modified Bessel functions of the first and second kind respectively of order

$$0, N = M \sqrt{(1 + \lambda_1)}, \quad \frac{dp_1}{dz} = (1 + \lambda_1) \frac{dp}{dz} \quad \text{and}$$

$G = (1 + \lambda_1) Gr$. The corresponding stream

function ($u = -\frac{1}{r} \frac{\partial \psi}{\partial z}$ and $w = \frac{1}{r} \frac{\partial \psi}{\partial z}$) is

$$\begin{aligned} \psi(r, z) = & \frac{1}{2C_{11}N^3} \frac{dp_1}{dx} \left\{ C_{11}N (r_1^2 - r^2) \right. \\ & + 2C_{14} (rI_1(Nr) - r_1I_1(Nr)) \\ & - 2C_{15} (rK_1(Nr) + r_1K_1(Nr_1)) \left. \right\} \\ & + \frac{1}{16C_{11}C_{13}N^3} \left\{ C_{25} (r^2 - r_1^2) \right. \\ & + C_{26} (rI_1(Nr) - r_1I_1(Nr_1)) \\ & + C_{27} (rK_1(Nr) - r_1K_1(Nr_1)) \\ & + C_{11}NG \left[2r^2 (4 + C_{12}\beta) \log r \right. \\ & + \log r_1 \left(\beta (r_1^4 - r^4) + 2\beta r^2 r_2^2 - C_{28} \right) \\ & \left. \left. + \log r_2 (r_1^2 - r^2) (8 + \beta (r_1^2 - r^2)) \right] \right\} \quad (20) \end{aligned}$$

where I_1 and K_1 are modified Bessel function of first order, first and second kind, respectively.

The instantaneous flow rate $Q(z)$ is given by

$$Q(z) = 2 \int_{r_1}^{r_2} r w(r, z) dz = \frac{dp_1}{dz} C_{24} + C_{23} \quad (21)$$

From equation (21) we get

$$\frac{dp_1}{dz} = \frac{1}{C_{24}} [Q(z) - C_{23}] \quad (22)$$

where the constants in Eqs. (19)- (22) are set in Appendix A.

Following the analysis given by Shapiro et al. [7], the mean volume flow \bar{Q} over a period is obtained as

$$\bar{Q} = Q(z) + \left(1 + \frac{\phi^2}{2} \right) - \epsilon^2 \quad (23)$$

Which on using Eq.(22) yields

$$\frac{dp}{dz} = \frac{1}{(1 + \lambda_1)C_{24}} \left[\bar{Q} + \epsilon^2 - \left(1 + \frac{\phi^2}{2} \right) - C_{23} \right] \quad (24)$$

The pressure rise Δp and the friction force (at the wall) on the outer and inner tubes are $F^{(o)}$ and $F^{(i)}$ respectively, in a tube of length L in their non-dimensional forms, are given by

$$\Delta p = \int_0^1 \frac{dp}{dz} dz \quad (25)$$

$$F^{(o)} = \int_0^1 r_2^2 \left(-\frac{dp}{dz} \right) dz \quad (26)$$

$$F^{(i)} = \int_0^1 r_1^2 \left(-\frac{dp}{dz} \right) dz \quad (27)$$

Substituting from Eq.(24) in Eqs. (25)- (27) with $r_1 = \epsilon, r_2 = 1 + \phi \cos(2\pi z)$ we get the pressure rise and the friction force (at the wall) on the outer and inner tubes.

Results and Discussions

MATHEMATICA is used to evaluate the quantitative effects of different parameters in the present analysis. Numerical values are obtained and plotted for different physical parameters of interest such as $\Delta P, F^{(o)}, F^{(i)}$ for different values of $\epsilon = 0.1$ upto 0.5, $M > \sqrt{2}, Gr = 0$ up to 9 and $\beta < 0$ (heat sink), $\beta = 0$ (absence heat source) or $\beta > 0$ (presence of heat source).

Fig.2 shows the variation of pressure rise versus the free convection parameter Gr for different values of β , the heat source/sink parameter. It is observed that the variation of pressure rise is linear with the free convection parameter. It is also observed that the increase in free convection parameter reduces the pressure rise in the presence of heat sink ($\beta = -15 < 0$). Conversely in the absence or presence of heat source ($\beta = 0$ or 5) it is observed that the increase in the free convection parameters increases the pressure rise

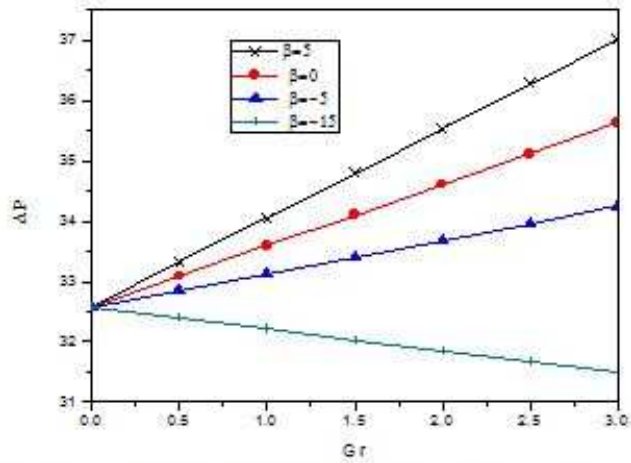


Fig.2. ΔP vs Gr for different values of β at $\phi = 0.4, \bar{Q} = 0.3, M = 4, \varepsilon = 0.3, \lambda = 0.2$

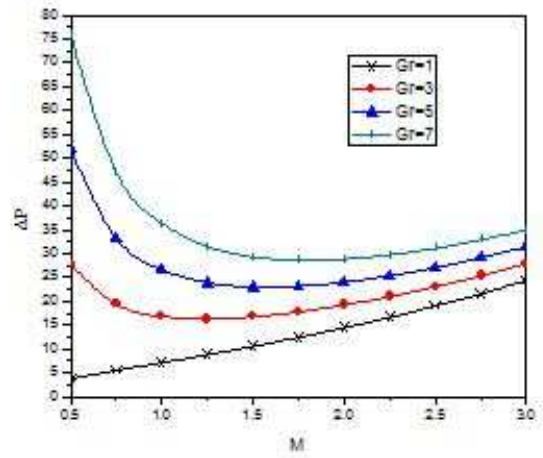
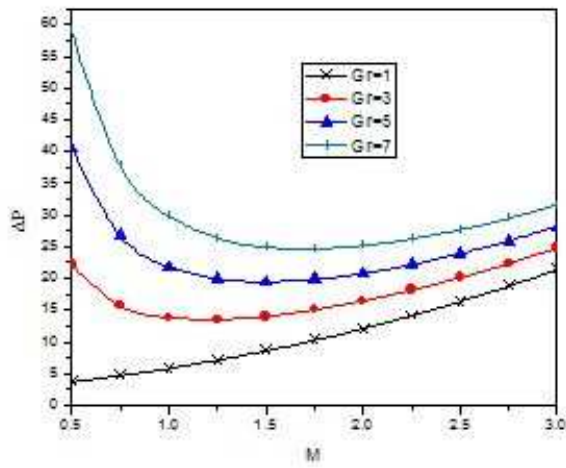


Fig.3. ΔP vs M for different values of Gr at Fig.4. ΔP vs M for different values of Gr at

$\phi = 0.4, \bar{Q} = 0.3, \beta = 5, \varepsilon = 0.1, \lambda = 0.2$

$\phi = 0.4, \bar{Q} = 0.3, \beta = 5, \varepsilon = 0.3, \lambda = 0.2$

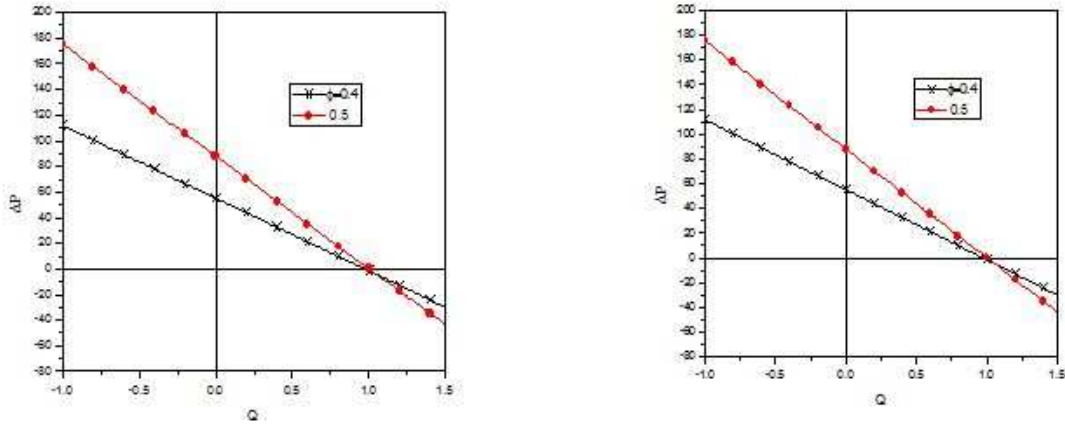


Fig.5. ΔP vs \bar{Q} for different values of ϕ at Fig.6. ΔP vs \bar{Q} for different values of ϕ at

$\phi = 0.4, Gr = 3, M = 4, \beta = 5, \varepsilon = 0.3, \lambda = 0.2$

$\phi = 0.4, Gr = 3, M = 4, \beta = 5, \varepsilon = 0.4, \lambda = 0.2$

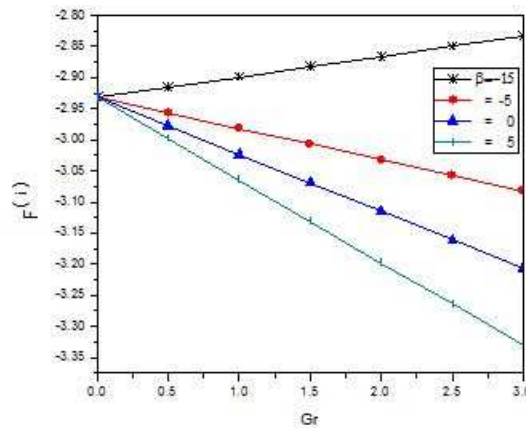


Fig.7 $F^{(1)}$ vs Gr for different values of β at $\phi = 0.4, \bar{Q} = 0.3, M = 4, \varepsilon = 0.3, \lambda = 0.2$

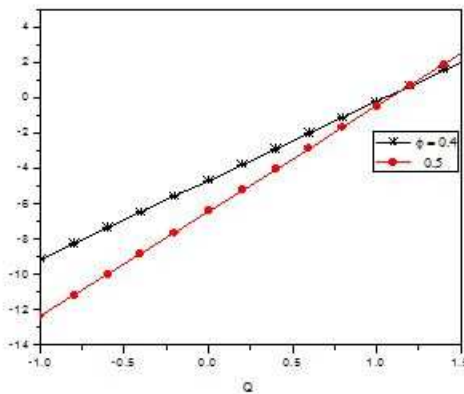


Fig.8. $F^{(1)}$ vs \bar{Q} for different values of ϕ at
 $Gr = 3, M = 4, \beta = 5, \varepsilon = 0.3, \lambda = 0.2$

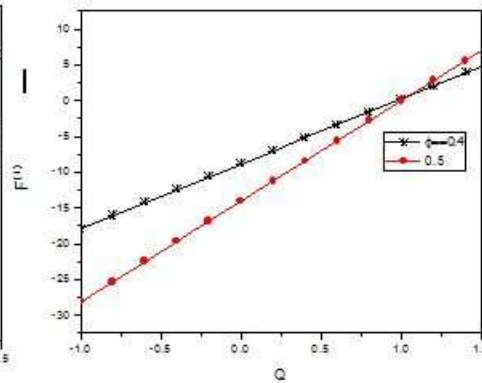


Fig.9. $F^{(1)}$ vs \bar{Q} for different values of ϕ at
 $Gr = 3, M = 4, \beta = 5, \varepsilon = 0.4, \lambda = 0.2$

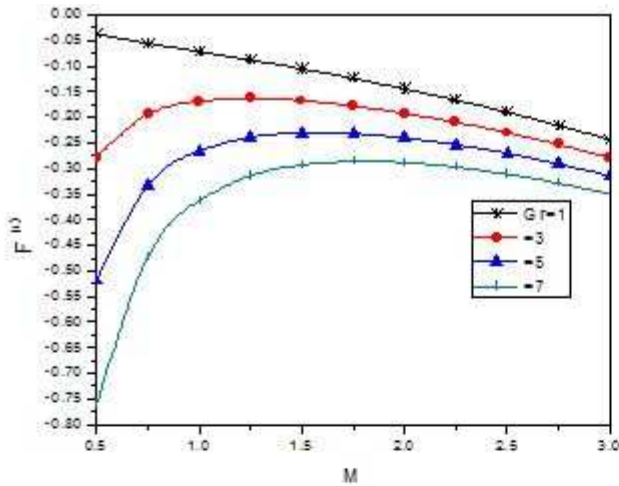


Fig10. $F^{(i)}$ Vs M for different values of Gr at

$$\bar{Q} = 0.3, \phi = 0.4, \beta = 5, \varepsilon = 0.1, \lambda = 0.2$$

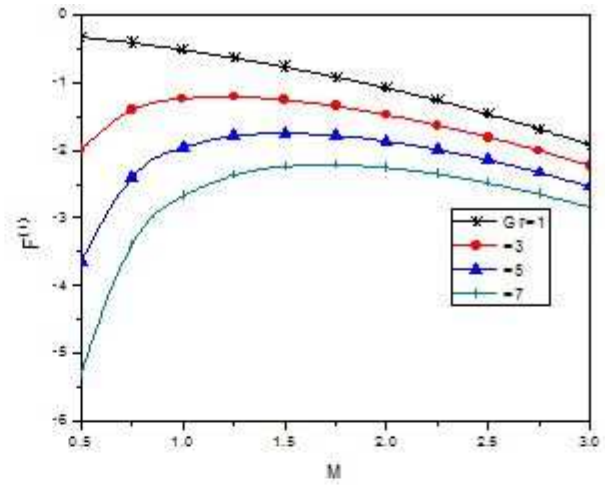


Fig.11 $F^{(i)}$ Vs M for different values of Gr at

$$\bar{Q} = 0.3, \phi = 0.4, \beta = 5, \varepsilon = 0.3, \lambda = 0.2$$

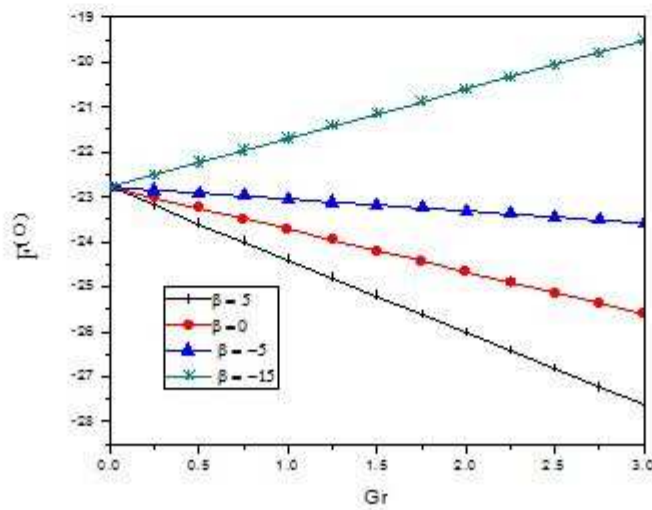


Fig.12. $F^{(0)}$ Vs Gr for different values of β at $\phi = 0.4, \bar{Q} = 0.3, M = 4, \varepsilon = 0.3, \lambda = 0.2$

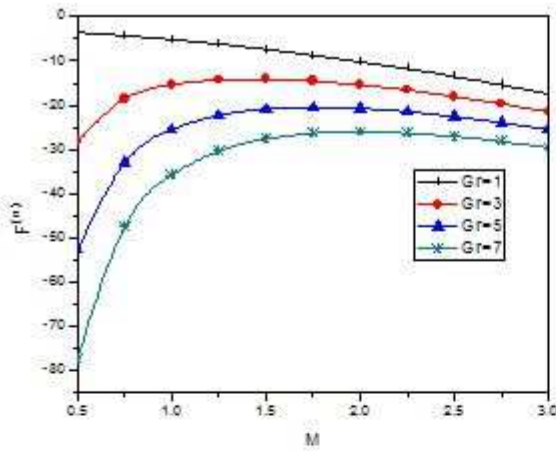


Fig.13. $F^{(0)}$ Vs M for different values of Gr at $\phi = 0.4, \bar{Q} = 0.3, \beta = 5, \epsilon = 0.1, \lambda = 0.2$

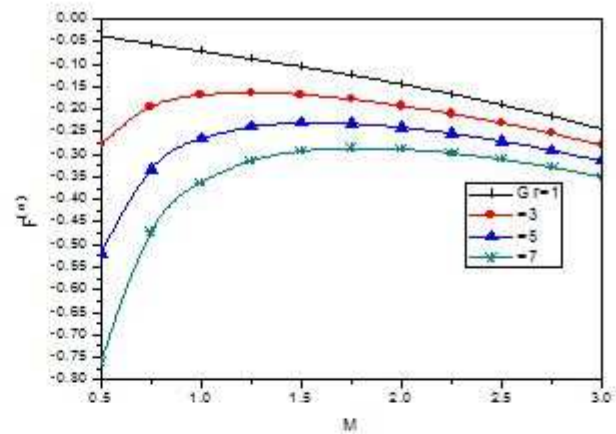


Fig.14. $F^{(0)}$ Vs M for different values of Gr at $\phi = 0.4, \bar{Q} = 0.3, \beta = 5, \epsilon = 0.3, \lambda = 0.2$

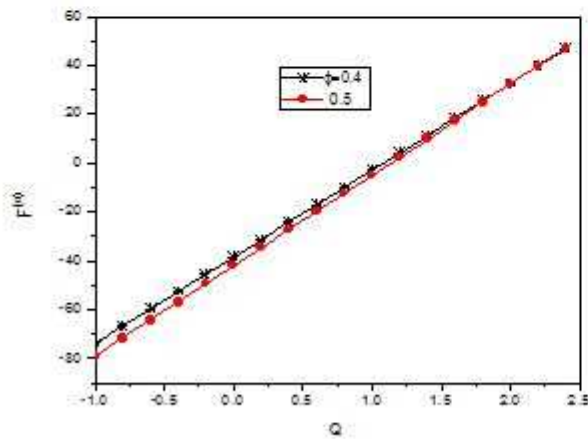


Fig.15. $F^{(0)}$ Vs \bar{Q} for different values of ϕ at $Gr = 3, M = 4, \beta = 5, \epsilon = 0.3, \lambda = 0.2$

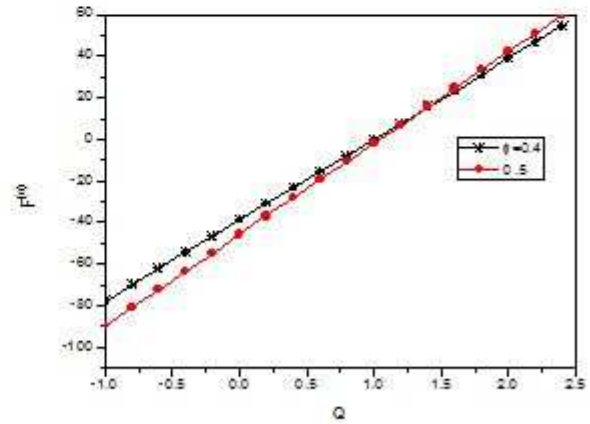


Fig.16. $F^{(0)}$ Vs \bar{Q} for different values of ϕ at $Gr = 3, M = 4, \beta = 5, \epsilon = 0.4, \lambda = 0.2$

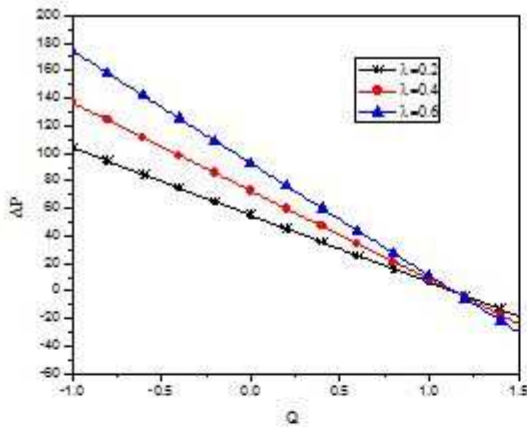


Fig.17 ΔP vs \bar{Q} for different values of λ at $\phi = 0.4, Gr = 3, M = 4, \beta = 5, \varepsilon = 0.1$

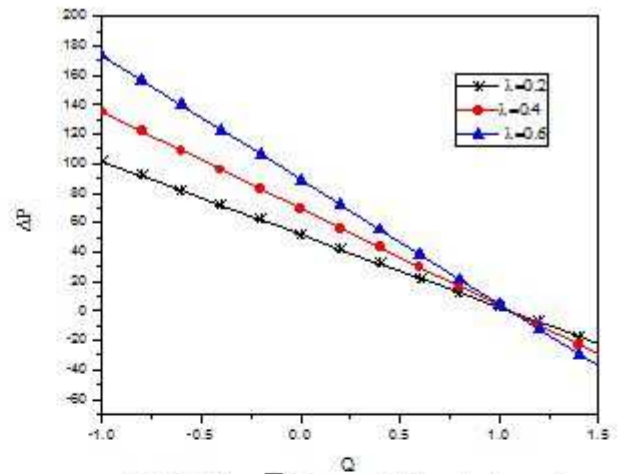


Fig.18. ΔP vs \bar{Q} for different values of λ at $\phi = 0.4, Gr = 3, M = 4, \beta = 5, \varepsilon = 0.3$

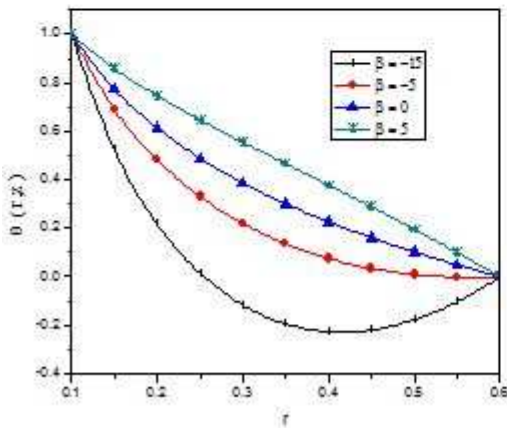


Fig.19. $\theta(r, z)$ vs r for different values of β at $\phi = 0.4, \varepsilon = 0.1, z = 0.5$

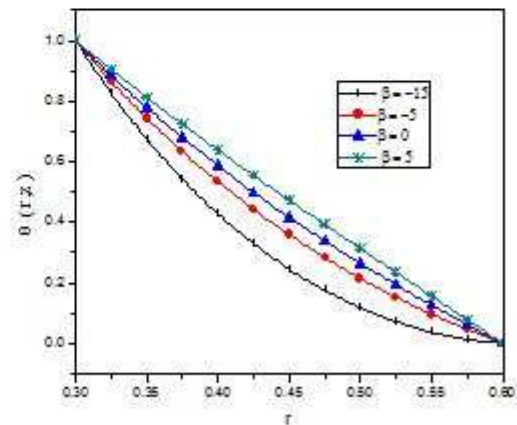


Fig.20. $\theta(r, z)$ vs r for different values of β at $\phi = 0.4, \varepsilon = 0.3, z = 0.5$

The Figs 3 & 4 show the variation of pressure rise with respect to magnetic parameter M for different values of ε and Gr . It is observed that the increase in M increases the pressure rise when the free convection parameter $Gr=1$. When $Gr=3,5$ and 7 the pressure rise decreases for smaller values of M , but for higher values

of M it increases. The figs also show that the pressure rise increases with increase in values of ε .

Figures 5 & 6 show the variation of pressure rise with respect to mean volume flow rate \bar{Q} for different values of ϕ and ε . It is observed that the pressure rise varies linearly with flow

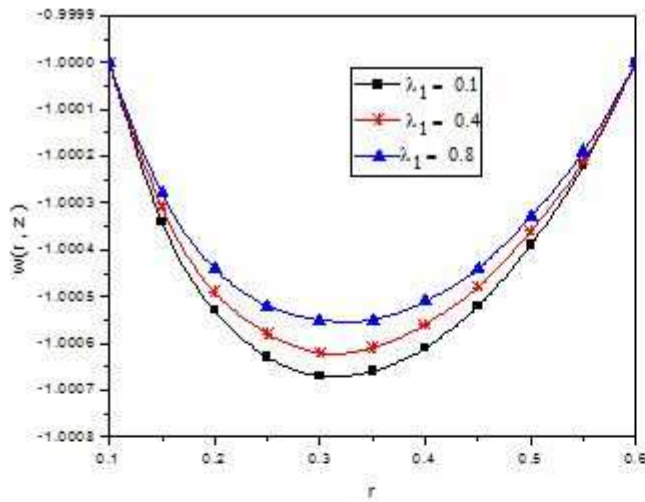


Fig.21. $w(r, z)$ vs r for different values of λ_1 at $\phi = 0.4, \varepsilon = 0.1, z = 0.5$

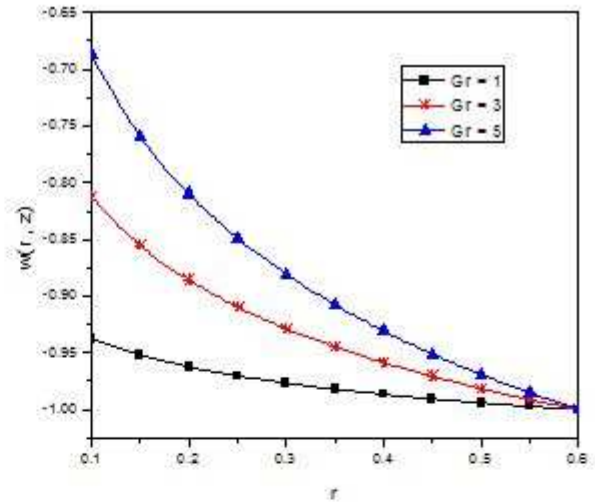


Fig.22 $w(r, z)$ vs r for different values of Gr at $\phi = 0.4, \varepsilon = 0.1, z = 0.5$

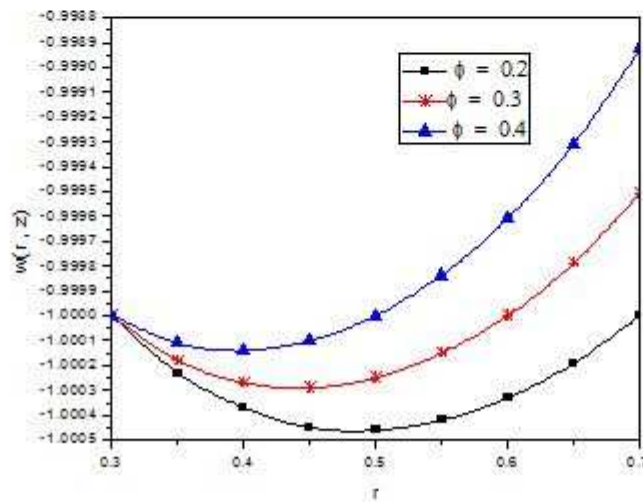


Fig.21. $w(r, z)$ vs r for different values of ϕ at $\phi = 0.4, \varepsilon = 0.1, z = 0.5$

rate. It is also observed that increase in the flow rate reduces the pressure rise. Pressure rise is observed to increase with increase in amplitude ratio ϕ . The pumping regions, peristaltic pumping ($\bar{Q} > 0$ & $\Delta P > 0$), augmented pumping ($\bar{Q} > 0$ & $\Delta P < 0$) and retrograde pumping

($\bar{Q} < 0$ & $\Delta P > 0$) are also shown if Figs 5 & 6. It is clearly seen that peristaltic pumping region becomes wider as the radius ε and amplitude ratio ϕ increase. In Fig.7 the variation of friction force on inner tube ($F^{(i)}$) versus the free convection parameter Gr is shown. It is seen that the variation of friction force on the inner tube is in opposite direction to that of pressure rise. Variation of friction force on inner tube versus

mean volume flow rate \bar{Q} is shown in figures 8 and 9 for different values ϕ . Figures 10 and 11 show the variation of friction force on inner tube versus magnetic parameter M . The friction force on the inner tube is exhibiting an opposite behavior to that of pressure rise.

Fig.12 shows the variation of friction force on the outer tube $F^{(o)}$ versus the free convection parameter Gr for different values of heat source/sink parameter β . Figures 13 and 14 show the variation of friction force on the outer tube with M , the magnetic parameter for different values of Gr and ε . Figures 15 and 16 show the variation of friction force on the outer tube versus the flow rate \bar{Q} . The friction force on the outer tube is observed to follow the friction force on the inner tube for variations of different parameters.

Figs 17 and 18 show the variation of pressure rise versus the free flow rate \bar{Q} for different values of the Jeffrey parameter λ_1 . It is observed that the pressure rise varies linearly with respect to flow rate. It is also observed that the increase in Jeffrey parameter λ_1 increases the pressure rise. However it is observed that the change in the radius ε has very small or negligible effect on the pressure rise pattern.

Figures 21 to 23 show the variation of velocity versus radius r for different values of Jeffrey parameter λ_1 , Gr and amplitude ratio. Figures show that the velocity increases with increase in Jeffrey parameter. Gr and the amplitude ratio.

Appendix A

$$C_{11} = I_0(Nr_2)K_0(Nr_1) - I_0(Nr_1)K_0(Nr_2), \quad C_{12} = r_1^2 - r_2^2,$$

$$C_{13} = \log \frac{r_1}{r_2}, \quad C_{14} = K_0(Nr_1) - K_0(Nr_2),$$

$$C_{15} = I_0(Nr_2) - I_0(Nr_1), \quad C_{16} = C_{11}C_{13},$$

$$C_{17} = r_1I_1(Nr_1) - r_2I_1(Nr_2),$$

$$C_{18} = r_1K_1(Nr_1) - r_2K_1(Nr_2),$$

$$C_{19} = C_{17}K_0(Nr_2) - C_{18}I_0(Nr_2),$$

$$C_{21} = 8NC_{12}C_{13}C_{11} - 16C_{17}C_{13}C_{14} + 16C_{18}C_{13}C_{15},$$

$$C_{22} = GNC_{12}C_{11}(4 + C_{14}\beta) - 8NC_{12}C_{11}(Gr^2 - N^2C_{12}) - GC_{13}\beta(r_1^4 - r_2^4) + 16C_{19}$$

$$C_{23} = \frac{C_{22}}{8N^3C_{13}C_{11}}, \quad C_{24} = \frac{C_{21}}{8N^3C_{13}C_{11}}, \quad C_{25} = -NC_{11}(8N^2C_{13} + G(4 + C_{14}\beta))$$

$$C_{26} = 16GC_{13}K_0(Nr_2), \quad C_{27} = 16GC_{13}I_0(Nr_2), \quad C_{28} = 2r_1^2(4 + (C_{12} + r_2^2)\beta)$$

References

[1] H.F. Bowman, Heat trans. Med. Biol.1 (1985) 193
 [2] S.B.Field,C. Francony, Physics and Technology of Hyperthermia, Martinus Nijhoff Publishers, Dordrecht,1987
 [3] D.P.Aikman, W.P. Anderson, Ann. Botany 35 (1971) 761

[4] M.J. Canny, O.M.Phillips, Ann. Botany 27 (1963) 379
 [5] V. Radhakrishnamurthy, G.Radhakrishnamacharya, P. Chandra. Advances in Phusiological Fluid Dynamics, Narosa Publishing House, India,1995
 [6] K. Vajravelu, G. Radhakrishnamacharya, Int. J. Non-Linear Mech. 42(2007) 754
 [7] Kh. S. Mekheimer, Y. Abd Elmaboud, Physics Letters. A 372 (2008) 1657-1665
 [8] A.H.Shapiro,M.Y. Jaffrin, S.L. Weinberg, J. Fluid Mech. 37 (1969) 799
 [9] R.A. Ramachandra, S. Usha. J. Fluid Mech 298 (1995) 271
 [10]Kh. S. Mekheimer, E.F.El Shehawey, A.M. Elaw. Int.J. Theor. Phys. 37 (1998) 2895
 [11]Kh. S. Mekheimer, J.Porous Media (2003) 189
 [12]Kh. S. Mekheimer, T.H. Al-Arabi. Int.J.Math. Sci. 26 (2003)1663
 [13]A.E.H.A. El Naby, A.E.M. El Misery, M. F. Abd El Kareem, Physica A 367 (2006)79
 [14]T.W. Latham, Fluid motion in a peristaltic pump, M. Sc. Thesis, MIT, Cambridge, MA, 1996
 [15]S. Takabatake. K.Ayukawa. J. Fluid Mech. 122 (1982) 439
 [16]S. Takabatake. K.Ayukawa, A. Mori, J. Fluid Mech. 193 (1988) 267
 [17]T.D. Brown, T.K Hung, J. Fluid Mech. 83 (1977) 249
 [18]D. Tang, M. Shen. J. Math. Anal. Appl. 174 (1) (1993)265
 [19]B.V. Rathish Kumar, K.B.Naidu, Comput. Fluids 24 (1995)161
 [20]T. Hayat, Masood Khan, A. M. Siddiqui, Asghar, Commun. Nonlinear Sci. Numer.Simul. 12 (2007) 910
 [21]V.K.Stud, G.S.Sephon, R.K Mishra, Bull. Math. Biol. 39 (1977) 385
 [22]H.L. Agrawal, B. Anwaruddin, Ranchi University Math. J. 15 (1984) 111
 [23]T. Hayat, E. Momoniat, F. M. Mahomed, Endoscope effect on MHD peristaltic flow of a power law fluid, Math. Problems Eng., doi: 10.1155/MPE/2006/84276
 [24]Kh. S. Mekheimer, Arab J. Sci.Eng.30 (1A) (2005) 69
 [25]T. Hayat, N. Ali, Physica A 370 (2006) 225
 [26]T. Hayat, N. Ali, Physica A 371 (2006) 188
 [27]F.M. Mahomed, T. Hayat, E. Momoniat, S. Asghar, Nonlinear Anal. Real world Appl. (2007) 853
 [28]T. Hayat, N. Ali, S. Asghar, Phys. Lett. A 363 (2007) 397

- [29] T. Hayat, N. Ali, S. Asghar, Acta. Mech. 193 (2007) 101
- [30] T. Hayat, N. Ali, Appl. Math. Comput. 188 (2007) 1491
- [31] N. Ali, T. Hayat, M. Sajid, Biotechnology 44 (2007) 125
- [32] T. Hayat, A. Afsar, M. Khan, S. Asghar, Comput. Math. Appl. 53 (2007) 1054
- [33] T. Hayat, N. Ali, S. Asghar, Appl. Math. Comput. 186 (2007) 309